

(29)  $\sum_{n=1}^{\infty} \frac{1}{n+1}$  ;  $\lim_{n \rightarrow \infty} \frac{1}{n+1} = 0$  which ~~is~~ diverges by  $n^{\text{th}}$  term test

(30)  $\sum_{n=1}^{\infty} \frac{2^n}{n+1}$  ;  $\lim_{n \rightarrow \infty} \frac{2^n}{n+1} = \infty \neq 0$  diverges by  $n^{\text{th}}$  term test.

(32)  $\sum_{n=1}^{\infty} \frac{1}{e^n}$  , geometric w/  $|r| = \frac{1}{e} < 1$  , converges  
or  $\lim_{n \rightarrow \infty} \frac{1}{e^n} = 0$  converges by  $n^{\text{th}}$  term test

(33)  $\sum_{n=1}^{\infty} \frac{2^n}{3^n+1}$  ;  $\frac{2^n}{3^n+1} < \left(\frac{2}{3}\right)^n$  .  $\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n$  converges

(geometric with  $|r| < 1$ )  $\sum_{n=1}^{\infty} \frac{2^n}{3^n+1}$  also converges  
by Direct Comparison Test.

(34)  $\sum_{n=1}^{\infty} n \cdot \sin\left(\frac{1}{n}\right)$  ;  $\lim_{n \rightarrow \infty} n \cdot \sin\left(\frac{1}{n}\right) \Rightarrow \lim_{u \rightarrow 0} \frac{1}{u} \cdot \sin(u)$   
let  $u = \frac{1}{n}$  ,  $n = \frac{1}{u}$  =  $\lim_{u \rightarrow 0} \frac{\sin u}{u} = 1$

$\sum_{n=1}^{\infty} n \cdot \sin\left(\frac{1}{n}\right) = 1 \neq 0$

$\therefore$  Diverges by  $n^{\text{th}}$  term test.

$$\textcircled{38} \sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^n, \quad \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e \neq 0$$

Diverges by  $n^{\text{th}}$  term test

$$\textcircled{39} \sum_{n=0}^{\infty} \frac{(-2)^n}{3^n} = \sum_{n=0}^{\infty} \left(\frac{-2}{3}\right)^n \text{ geometric series}$$

with  $|r| < 1$ , converges.